INDUCTIVE DRAG OF A CURVED WING

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UDC 629.735.33.015.3.025.1:
533.6.12/13

In aviation, wings formed by hinging the surfaces adjoining an axisymmetrical fuselage are used; it is desirable to select the angle ϕ at which the rotational axes of the wings are positioned and their aperture angle θ (Fig. 1) so as to minimize the inductive drag. The largest wing span is obtained at angles close to $\phi = \pi/3$, $\theta = \pi/2$, but, aside from the span, the curvature of the wing also affects the inductive drag. There is no adequately substantiated method for calculating the lift force and the inductive frag of the system consisting of the wing and a body of revolution; this is attributable to the fact that a body of revolution does not have a sharp training edge, fixing the trailing critical line, as in a wing. It is not known whether free vortices are shed by a body of revolution. If the root chord of the wing is large compared to the diameter of the fuselage, then it may be assumed that the velocity of circulation in the root section of the wing does not vanish, i.e., its effect (the difference in the pressure at the bottom and at the top of the fuselage) extends to the fuselage.

Let us assume that the medium is incompressible, the stern part of the fuselage is tapered, the angle of attack is $\alpha = 0$, the streaming flow does not separate, and in accordance with the linear theory in [1, 2] free vortices lie along the streamlines of the flow near the isolated fuselage, while near the wing the fuselage has the shape of a very long cylinder. Then the flow velocity in this region is close to the velocity of the unperturbed flow and the distance of the streamline from the axis of the cylinder r is related to this distance at infinity behind the fuselage r_∞ by the continuity equation

$$r^2 - R^2 = r_\infty^2 , \qquad (1)$$

where R is the radius of the cylinder.

Equation (1) enables finding the form of the transverse cross section of a free vortex layer at infinity behind the wing-fuselage system (in Treft's plane) from the form of the wing; in Fig. 1 the broken lines show the cross sections of the free vortex layers for two cases, differing by the angle ϕ. The lift force and the inductive drag of the wing-fuselage system are determined completely by the free vortex layer in the Treft plane. For this reason, Munk's theorem, which holds for all vortex systems, is applicable to the system. According to this theorem, in the presence of a fixed lift force the inductive drag of the system is minimum if the distribution of the circulation is such that the layer of free vortices descends with a constant velocity λV as a solid body [3] (V is the velocity of the unperturbed flow). To determine the effect of the curvature of the wing on the minimum inductive drag we shall study the case when the form of the transverse section of the free vortex layer is close to the form of the arc of a circle, as shown in Fig. 1.

The lift force and the inductive drag [3] are given by

$$Y = \rho V \int \Phi dx, \quad X = \frac{\rho}{2}\int \Phi \frac{\partial \Phi}{\partial n} ds = -\frac{\lambda}{2} Y.$$

Here Φ is the velocity potential; $\partial\Phi/\partial n = -\lambda V \cos(n, y)$ is the derivative of the potential along the direction of the inner normal to the contour of the free vortex layer (the layer is viewed as a section); s is the arc length of the contour of the layer; the integrals extend along the contour of the layer in Treft's plane; and, ρ is the density of the medium.

The arc of the circle (Fig. 2) is mapped into a circle with the help of the function

$$z = \zeta + c^2/\zeta,$$

Zhukovskii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 38-40, May-June, 1986. Original article submitted April 25, 1985.

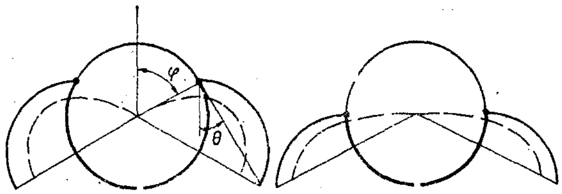

Fig. 1

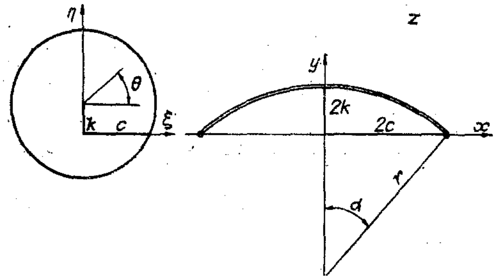

Fig. 2

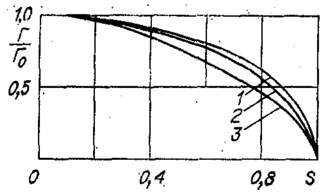

Fig. 3

and the coordinates of a point on the arc of the circle

$$x = \frac{2(c^2+k^2)\left(\sqrt{c^2+k^2} + k\sin\theta\right)\cos\theta}{c^2+2k^2+2k\sqrt{c^2+k^2}\sin\theta}, \tag{2}$$
$$y = \frac{2k\left(k+\sqrt{c^2+k^2}\sin\theta\right)^2}{c^2+2k^2+2k\sqrt{c^2+k^2}\sin\theta},$$

the radius of the arc $r = k + c^2/k$, and the angle $\alpha = \arcsin\,[2ck/(c^2+k^2)]$. The velocity potential on the circle

$$\Phi = -2\lambda V\sqrt{c^2+k^2}\sin\theta.$$

To calculate the lift force we transform to the independent variable θ and integrate by parts:

$$\int \Phi dx = \int_0^{2\pi} \Phi \frac{dx}{d\theta}\,d\theta = -\int_0^{2\pi} x\frac{d\Phi}{d\theta}\,d\theta = -4\lambda V(c^2+k^2)^{3/2},$$
$$\int_0^{2\pi} \frac{\left(\sqrt{k^2+c^2}+k\sin\theta\right)\cos^2\theta d\theta}{c^2+2k^2+2k\sqrt{c^2+k^2}\sin\theta} = -2\pi\lambda V(2c^2+k^2),$$

and therefore

$$Y = -2\rho\lambda V^2\pi(2c^2+k^2), \quad X = \frac{Y^2}{4\rho V^2\pi(2c^2+k^2)}.$$

The ratio of the inductive drag of a curved wing to that of a straight wing X_0 for the same lift forces and spans (in Treft's plane) has the form

$$\frac{X}{X_0} = \frac{1}{1+\frac{1}{2}\left(\frac{k}{c}\right)^2},$$

i.e., the drag of a curved wing is smaller than that of a straight wing, and the angles ϕ, θ must be chosen taking into account the effect of both the span and the curvature of the wing.

From the second formula of (2) it follows that

$$\sin\theta = -\frac{k}{\sqrt{c^2+k^2}}\left(1-\frac{y}{2k}\right) \pm \sqrt{\frac{y}{2k}\left[1-\frac{k^2}{c^2+k^2}\left(1-\frac{y}{2k}\right)\right]},$$

and since the circulation of the velocity Γ equals the discontinuity of the potential,

$$\Gamma = 4\lambda V\sqrt{c^2+k^2}\sqrt{\frac{y}{2k}\left[1-\frac{k^2}{c^2+k^2}\left(1-\frac{y}{2k}\right)\right]},$$

the tip of a curved wing is under a smaller load than the tip of a straight wing (Fig. 3, where 1-3 correspond to k/c = 0, 0.5, and 1).

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EFFECT OF SURFACE POTENTIAL AND INTRINSIC MAGNETIC FIELD ON RESISTANCE OF A BODY IN A SUPERSONIC FLOW OF RAREFIED PARTIALLY IONIZED GAS

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UDC 533.95:538.4:537.523:550.38

The character of flow over a body, structure of the perturbed zone, and flow resistance in a supersonic flow of rarefied partially ionized gas are determined by the intrinsic magnetic field and surface potential of the body. The effects of intrinsic magnetic field and surface potential were studied in [1-4]. There have been practically no experimental studies of the effect of intrinsic magnetic field on flow of a rarefied plasma. Studies of the effect of surface potential have been limited to the case $R/\lambda_d \lesssim 50$ [1, 3]; this is due to the difficulty of realization of flowover regimes at $R/\lambda_d \gtrsim 10^2$ (where R is the characteristic dimension of the body and λ_d is the Debye radius). At the same time $R/\lambda_d \gtrsim 10^2$, the regime of flow over a large body, is of the greatest practical interest. The present study will consider the effect of potential and intrinsic magnetic field on resistance of a large ($R/\lambda_d \gtrsim 10^2$) axisymmetric body (disk, sphere) in a supersonic flow of rarefied partially ionized gas.

1. Experiments were performed with a plasma aerodynamic tube in a flow of partially ionized gas produced by a gas discharge accelerator with the working material ionized by electron collision. The plasma flow with intensity $j_\infty \cong 10^{15} - 10^{17}$ cm^{-2} sec^{-1} was fed to the working chamber with residual gas pressure of $\sim 4\cdot 10^{-5}$ Pa. The plasma flow parameters in the working chamber at a pressure of $\sim 10^{-3}$ Pa were measured by movable electrostatic probes of three types: planar with working surface 3.5 mm in diameter, made of molybdenum, cylindrical probe in thermoanemometer form [5] with working section made of 0.06 mm tungsten wire 6.5 mm long, isolated molybdenum wire probe with diameter 0.04 mm and length 2.3 mm.

Dnepropetrosk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 41-47, May-June, 1986. Original article submitted April 17, 1985.

